ICFP M1 - Quantum Matter - TD n°6 - Exercises
Spinwaves in the antiferromagnetic Heisenberg model

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Consider the antiferromagnetic Heisenberg model for a cubic lattice of spin s, considering only nearest-neighbor exchange. De note N the number of sites in the lattice, d the dimension and put the lattice size $a = 1$.

1. Write down the Hamiltonian.
2. What is the ground state (albeit not exact) of this model?

   Using Holstein-Primakoff bosons we want to calculate the spin wave spectrum above this ground state.
3. Rewrite the spin vectors using Holstein-Primakoff bosons. Recall that we have two sublattices.
4. Go to momentum space (we still have two sublattices) and rewrite the Hamiltonian neglecting 4-order terms.

   Notice that what you obtained does not take the form of a simple harmonic oscillator Hamiltonian, but that it is quadratic and that fields with different $\mathbf{k}$ and $-\mathbf{k}$ are decoupled. In order to transform boson bilinears of the form $a^\dagger a^\dagger$ and aa into "normal" terms such as $a^\dagger a$ (or $aa^\dagger$), it is necessary to perform a Bogoliubov transformation, i.e. to define

$$\begin{cases} a_{\mathrm{A}\mathbf{k}} = (\cosh \eta_{\mathbf{k}}) b_{1\mathbf{k}} + (\sinh \eta_{\mathbf{k}}) b^\dagger_{2-\mathbf{k}} \\ a^\dagger_{\mathrm{B}-\mathbf{k}} = (\cosh \eta_{\mathbf{k}}) b^\dagger_{2-\mathbf{k}} + (\sinh \eta_{\mathbf{k}}) b_{1\mathbf{k}} \end{cases} \tag{1}$$

   We will choose $\eta_{\mathbf{k}}$ to simplify H
5. check that b and $b^\dagger$ satisfy canonical bosonic commutation relations, $[b_{l\mathbf{k}}, b^\dagger_{l\mathbf{k}}] = 1$ ($l = 1, 2$) etc.
6. Plug these expression into the Hamiltonian and find $\eta_{\mathbf{k}}$ such that all the "anormal" terms vanish. You can ignore constants.
7. Now find the dispersion relation at small k and plot it.
8. What are the main differences with the ferromagnetic spectrum?
9. Compute $\langle S^z_{i\in \mathrm{B}} \rangle - s$. This is the correction to the staggered magnetization (due to quantum fluctuations at $T = 0$).