TD 2: From Heat Engines to Tropical Cyclones

Baptiste Coquinot *Contact:* baptiste.coquinot@ens.fr *Webpage:* https://coquinot.fr

23 septembre 2021

Every years, there about 80 cyclones in world, the majority in the Pacific ocean and about 20 in the Atlantic ocean. Formed in the inter-tropical areas , they can be dangerous when they touch the land. Since 30 years, the cyclones are modelled as a heat machine working thanks the difference of temperature between the ocean and the troposphere. This exercise is based on the work of Kerry Emanuel (https://emanuel.mit.edu/).

The model is a presented in the scheme. The ocean is a thermal reservoir at $T_c = 300$ K, while the troposphere is a thermal reservoir at $T_f = 200$ K. The system is is made of a mass m_a of dry air and δm_w of water, which is a liquid or a gas depending on the steps of the process. The typical size of the system is $L = 10$ m. All the transformations are supposed reversible and we denote P_i, v_i, z_i the pressure, velocity and height at the point $i \in \{A, B, C, D\}$. We remind that the massive enthalpy of water vaporisation is $l_v = 2500\,\text{kJ}\cdot\text{kg}^{-1}$.

1. In a cyclic process, what physical quantity should be study? Write the variation of this quantity, in particular in function of the effective work W'.
2. Remind the thermodynamic identity of this physical quantity and the link between the variation of entropy S and the heat Q.

Process from A to B. The air is A is dry and moves to the cyclone's eye where the pressure is low. The interaction with the ocean have two consequences. First, the temperature is kept at T_c. Second, the mass δm_w gets vaporised and mixes with the air. At this point, $\delta m_w / m_a \approx 5 \cdot 10^{-3} \ll 1$.

3. Express the variation of enthalpy ΔH_{AB}, entropy ΔS_{AB}, the effective work W'_{AB} and the heat Q_{AB}. We use the approximation $v_A \ll v_B$.
4. We will see that $W'_{AB} < 0$, what does it represent physically?

Process from B to C. The air is in the cyclone's eye and moves up adiabatically. During this ascension, the water becomes liquid and falls in the ocean (it's raining). At the point C, the air has reached the temperature T_f and is close to mechanical equillibirum, *i.e.* $v_c \ll v_B$. There is no work during this process.

5. Give orders of magnitude of the ascension time, the diffusion time and the machanical time to justify that the process is adiabatic.
6. Express the variation of enthalpy ΔH_{BC}, entropy ΔS_{BC}, the effective work W'_{BC} and the heat Q_{BC}.
7. Calculate the height z_C. What is the role of the vapour of water?

Process from C to D. The air is in the troposphere at T_f and transfer heat to the space by radiation. Its height decreases to $z_D < z_C$. Since v_B is the dominant velocity, we may set $v_C = v_D$.

8. Express the change of pressure as a function of the change of height.
9. Express the variation of enthalpy ΔH_{CD}, entropy ΔS_{CD}, the effective work W'_{CD} and the heat Q_{CD}.

Process from D to A. The air is going back to its initial state. Again, this process is adiabatic.

10. Express the variation of enthalpy ΔH_{DA}, entropy ΔS_{DA}, the effective work W'_{DA} and the heat Q_{DA}.
11. Deduce $z_D - z_C$.

Conclusion. We now make global balance equations.

12. Express v_B. We may introduce η the Carnot yield.
13. Given that for a large hurricane, the depression at the eye may be of the order $P_B \sim 10^4$ Pa, give an estimate of v_B.
14. We define the yield of the cyclone as $\eta^* = -\frac{W'}{Q_{AB}}$. Give its expression.

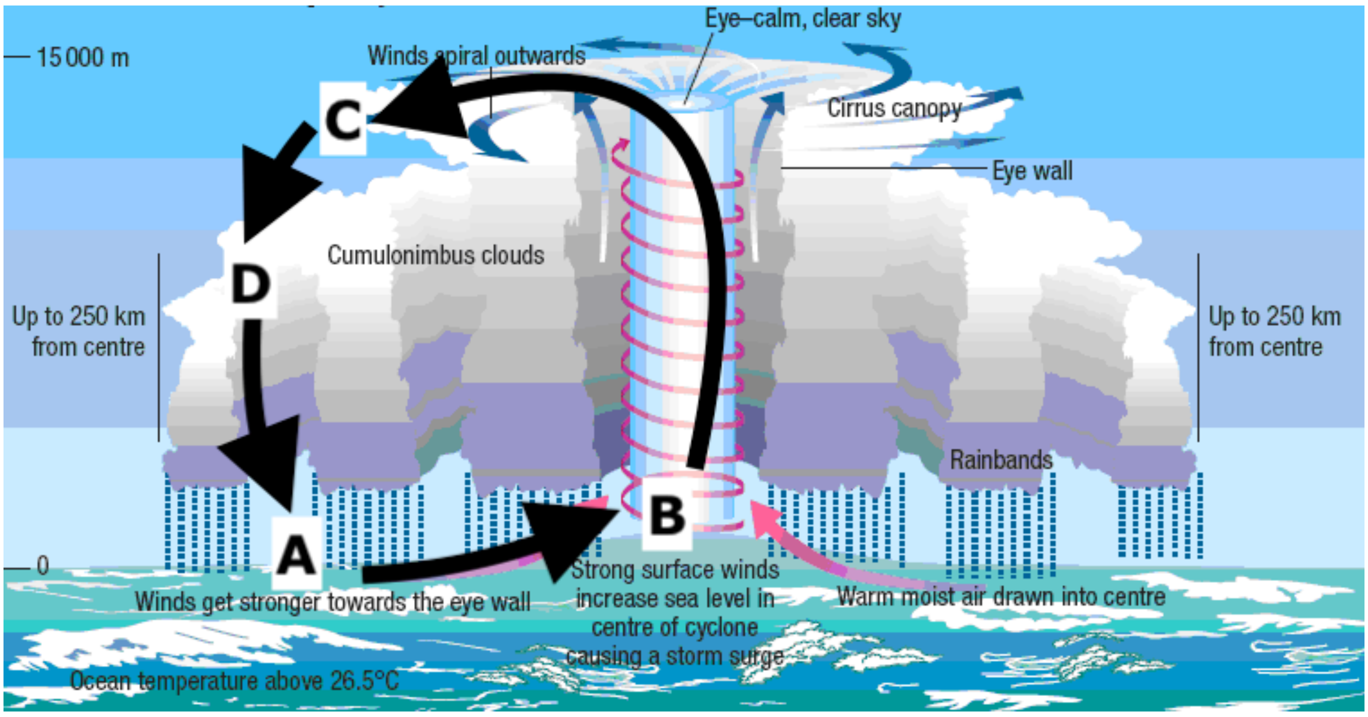

Figure 1: Schematic of the cyclone.

15. What is the effective mechanical power $\mathcal{P}'$ generated by the cyclone? We will introduce $\mathcal{D}$ the massive flow rate.

16. In which form is converted this energy? Given that $\mathcal{P}' \sim 10^{13}$ W, give an estimate of $\mathcal{D}$.

——————— *Only when you have finished all the exercises* ———————

The Wikipedia Moment. SADI CARNOT (1796-1832).

Sadi Carnot was born in Paris at the Palais du Petit-Luxembourg into a family that was distinguished in both science and politics. He was the first son of Lazare Carnot, an eminent mathematician, military engineer, and leader of the French Revolutionary Army. Lazare chose his son's third given name (by which he would always be known) after the Persian poet Sadi of Shiraz. Sadi was the elder brother of statesman Hippolyte Carnot and the uncle of Marie Frans Sadi Carnot, who would serve as President of France from 1887 to 1894.

In 1811, at the age of 16, Sadi Carnot became a cadet in the ole Polytechnique in Paris. The ole Polytechnique was intended to train engineers for military service, but its professors included such eminent scientists as Andrarie Amp, Frans Arago, Joseph Louis Gay-Lussac, and Sim Denis Poisson. Thus, the school had become renowned for its mathematical instruction.

After graduating in 1814, Sadi became an officer in the French army's corps of engineers. His father Lazare had served as Napoleon's minister of the interior during the "Hundred Days", and, after Napoleon's final defeat in 1815, Lazare was forced into exile. Sadi's position in the army, under the restored Bourbon monarchy of Louis XVIII, became increasingly difficult. Sadi Carnot was posted to different locations, where he inspected fortifications, tracked plans, and wrote many reports. It appeared that his recommendations were ignored and his career was stagnating.

In 1819, Sadi transferred to the newly formed General Staff in Paris. He remained on call for military duty, but from then on he dedicated most of his attention to private intellectual pursuits and received only two-thirds pay. He became interested in understanding the limitation to improving the performance of steam engines, which led him to the investigations that became his Reflections on the Motive Power of Fire, published in 1824.

Carnot's book received very little attention from his contemporaries. The only reference to it within a few years after its publication was in a review in the periodical Revue Encyclopque, which was a journal that covered a wide range of topics in literature. The impact of the work had only become apparent once it was modernized by ile Clapeyron in 1834 and then further elaborated upon by Clausius and Kelvin, who together derived from it the concept of entropy and the second law of thermodynamics. Rankine, who introduced the term potential energy in 1853, was later made aware that an equivalent phrase, "in its purely mechanical sense, had been anticipated by Carnot", who had employed the term force vive virtuelle.

Carnot retired from the army in 1828, without a pension. He was interned in a private asylum in 1832 as suffering from "mania" and "general delirum", and he died of cholera shortly thereafter, aged 36, at the hospital in Ivry-sur-Seine.