TD 3: Thermoelectric Generator

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In this exercise we consider a thermo-electric generator, *i.e.* we produce an electrical power from a gradient of temperature. Our model is described in figure 1. The hot thermal reservoir provides a heat flux $\dot{Q}_c$ while the cold thermal reservoir obtains a heat flux $\dot{Q}_f$. The electric circuit is a double P-N junction. Each semiconductor has its own resistivity $\rho_{P/N}$ and thermal conductivity $k_{P/N}$. In the first two sections of the exercise we only consider one of the semiconductor separately from the other. We denote $\mathcal{I}$ the electric current. The effective electric power $\mathcal{P}$ can be used to power an engine. We model it by a resistance R_L.

1. What is the most common way to produce electricity from a hot thermal reservoir?

Joule effect. We consider Joule effect to study $\dot{Q}(x)$ inside one of the semiconductors.

2. What is the physical origin of Joule effect?
3. Apply Joule effect to provide an expression for $\dot{Q}(x)$.

Fourier's law. This heat flux distribution must match with Fourier's law.

4. What is the physical origin of Fourier's law? Give its usual form.
5. Deduce $\dot{Q}(0)$ and $\dot{Q}(L)$.
6. Give the distribution of temperature. Comment.

Peltier effect. We now consider the P-N junctions.

7. Remind the Peltier effect.
8. Denoting $\alpha = \alpha_P - \alpha_N$, give the expression of $\dot{Q}_c$ and $\dot{Q}_f$.

Power and efficiency. We now consider the global system and denote $\rho = \rho_N + \rho_P$ and $k = k_N + k_P$.

9. Relate the electric current $\mathcal{I}$ with the resistance R_L and give the electrical power $\mathcal{P}$.
10. Give the expression of the efficiency $\eta = \frac{\mathcal{P}}{\dot{Q}_c}$. In which limit do we find Carnot efficiency η_C? Is it physically plausible?
11. What is the free parameter in the system? What is the maximal value of $\mathcal{I}$ denoted $\mathcal{I}_m$.
12. Show that

$$\eta = \eta_C \frac{j(1-j)}{j - \frac{1}{2}\eta_C j^2 + A} \tag{1}$$

where $j = \frac{\mathcal{I}}{\mathcal{I}_m}$, η_C is the Carnot efficiency and $A = \frac{KR}{\alpha^2 T_C}$.

Power-efficiency diagram. We now study the efficiency and power as a function of j.

13. Calculate the maximal power $\mathcal{P}_m$ and the associated current. Deduce that

$$\frac{\mathcal{P}}{\mathcal{P}_m} = 4j(1-j). \tag{2}$$

14. Calculate the current j^* at maximal efficiency. We will assume $A \gg 1$.
15. Draw the diagram $\left(\frac{\mathcal{P}}{\mathcal{P}_m}, \eta\right)$. Comment.

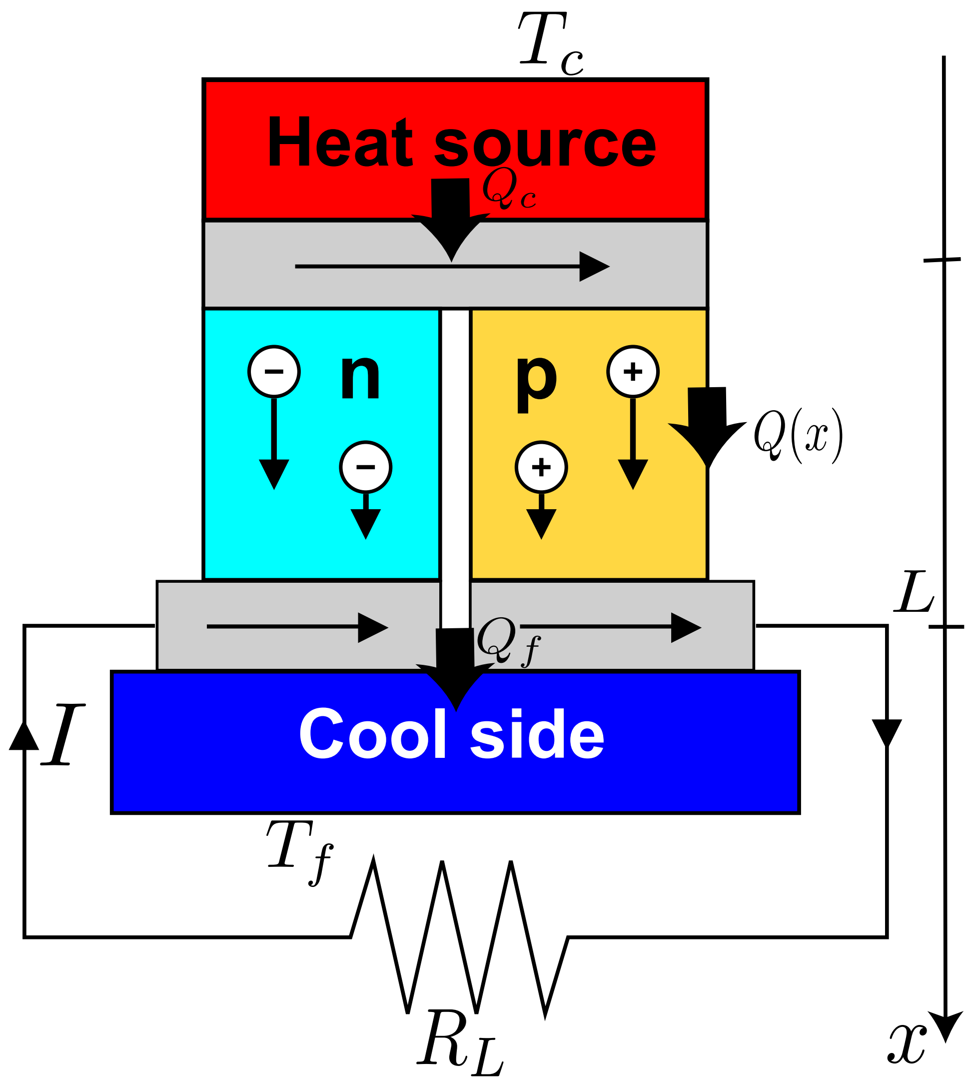

Figure 1: Schematic of the thermoelectric generator.

Only when you have finished all the exercises

The Wikipedia Moment. ÉMILE CLAPEYRON (1799-1864).

Born in Paris, Clapeyron studied at the École polytechnique, graduating in 1818. He also studied at École des mines. In 1820 he and Gabriel Lamé went to Saint Petersburg to teach and work at the school of public works there. He returned to Paris only after the Revolution of July 1830, supervising the construction of the first railway line connecting Paris to Versailles and Saint-Germain and took his steam engine designs to England in 1836.

In 1834, he made his first contribution to the creation of modern thermodynamics by publishing a report entitled Mémoire sur la puissance motrice de la chaleur (Memoir on the Motive Power of Heat), in which he developed the work of the physicist Sadi Carnot, deceased two years before. Though Carnot had developed a compelling analysis of a generalised heat engine, he had employed the clumsy and already unfashionable caloric theory. Clapeyron, in his memoire, presented Carnot's work in a more accessible and analytic graphical form, showing the Carnot cycle as a closed curve on an indicator diagram, a chart of pressure against volume (named in his honor Clapeyron's graph). Clapeyron's analysis of Carnot was more broadly disseminated in 1843 when Johann Poggendorff translated it into German.

In 1842 Clapeyron published his findings on the "optimal position for the piston at which the various valves should be opened or closed." In 1843, Clapeyron further developed the idea of a reversible process, already suggested by Carnot and made a definitive statement of Carnot's principle, what is now known as the second law of thermodynamics. These foundations enabled him to make substantive extensions of Clausius' work, including the formula, now known as the Clausius–Clapeyron relation, which characterises the phase transition between two phases of matter. He further considered questions of phase transitions in what later became known as Stefan problems. Clapeyron also worked on the characterisation of perfect gases, the equilibrium of homogeneous solids, and calculations of the statics of continuous beams, notably the theorem of three moments (Clapeyron's theorem).

Clapeyron married Mélanie Bazaine, daughter of Pierre-Dominique Bazaine (mathematician and ingénieur des ponts), and older sister of Pierre-Dominique Bazaine (railway engineer) and Francois Achille Bazaine (Marshal of France). He died in 1864.