# TD 6: Thermodynamics of Climate and Renewable Energies - Solutions

Baptiste Coquinot *Contact:* baptiste.coquinot@ens.fr *Webpage:* https://coquinot.fr

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**1 Solar Power** In this exercise we estimate the Solar power and what energy we can extract.

1. By comparing the heat you feel from a 100 W light bulb and from the Sun, give an order of magnitude of the power $\mathcal{P}_S$ emitted by the Sun.
2. By dimensional analysis, find the Wien law which relates the temperature of the Sun $T_S$ and the main wave length of emission $\lambda_m$.
3. By dimensional analysis, find the Stephan-Boltzmann law which relates the temperature of the Sun $T_S$ and the emission power $\mathcal{P}_S$.
4. The surface temperature of the Sun is $T_S \approx 6 \cdot 10^3$ K and its radius is $R_S \approx 7 \cdot 10^8$ m. Given that Stephan's constant is $\sigma \approx 5.7 \cdot 10^{-8}$ usi, deduce $\mathcal{P}_S$.
5. Deduce the power received by the Earth $\mathcal{P}_{SE}$ and the power received by surface unit $p_{ST}$. For the latitude of France, $p_{ST}$ ranges between 100 and 150 W/m$^2$.
6. Considering that we cover 1% of France surface by solar panels, what is the power received?
7. You have seen that the PN junction has a maximal efficiency which is the Shockley-Queisser limit and is about 30%. Taking this into account, what is the maximal solar power we may imagine to extract? For comparison, the energy consumption in France is $\approx 300$ GW and the solar energy production is $\approx 1$ GW.
8. Considering that 10% of the surface of the Earth is made of plants and that photosynthesis has an efficiency of $\approx 10\%$, estimate the power obtained by photosynthesis.
9. What is the need of energy to provide food to the whole population. For comparison the agriculture provide typically $\sim 10^{12}$ W and the total resources in biomass ares estimated to $\sim 10^{13}$ W.

**Correction**

1. The heat is similar when you are at $d \sim 10$ cm from the light bulb of power $\mathcal{P}_{LB} \sim 100$ W. Since the power is diving over the surface it reaches, and using that the Sun-Earth distance is $d_S \sim 10^{11}$ m, the power of Sun is typically of order
$$\mathcal{P}_S = \mathcal{P}_{LB} \left( \frac{d_S}{d} \right)^2 \sim 10^{26} \text{ W}. \quad (1)$$
Actually, $\mathcal{P}_S \approx 3.8 \cdot 10^{26}$ W.
2. The relevant quantities are $k_B T_S$, $c$, $\hbar$ and $\lambda_m$. We have 4 quantities and 3 dimensions (energy, length and time) thus 1 dimensionless number. We then conclude
$$\lambda_m \propto \frac{\hbar c}{k_B T_S}. \quad (2)$$
More precisely, $\lambda_m T_S \approx 2.9 \cdot 10^{-3}$ m $\cdot$ K.
3. The relevant quantities are $\mathcal{P}_S / \mathcal{A}_S$, $c$, $\hbar$ and $k_B T_S$ where $\mathcal{A}_S$ is the area of the surface of the Sun. We have 4 quantities and 3 dimensions (energy, length and time) thus 1 dimensionless number. We then conclude
$$\mathcal{P}_S \propto \mathcal{A}_S \frac{(k_B T_S)^4}{c^2 \hbar^3}. \quad (3)$$
More precisely, $mcP_S = \mathcal{A}_S \sigma T_S^4$ where $\sigma \approx 5.7 \cdot 10^{-8}$ usi.
4. We calculate $\mathcal{P}_S = 4\pi R_S^2 \sigma T_S^4 \approx 4.5 \cdot 10^{26}$ W.
5. We have $\mathcal{P}_{SE} = \frac{\mathcal{P}_S}{4\pi d_S^2} \pi R_E^2 \approx 1.7 \cdot 10^{17}$ W and $p_{SE} = \frac{\mathcal{P}_{SE}}{4\pi R_T^2} \approx 340$ W/m$^2$.
6. We cover a surface of $5 \cdot 10^9$ m$^2$, thus the received power is $\approx 5 \cdot 10^{11}$ W= 500 GW.
7. With this efficiency, the power falls to $\sim 150$ GW.
8. We directly get $10^{14}$ W.
9. A human need $\approx \sigma S(310^4 - 290^4) \approx 100$ W=2000kC/day to remain at the same temperature. Taking $10^{10}$ humans we find a need of $10^{12}$ W.

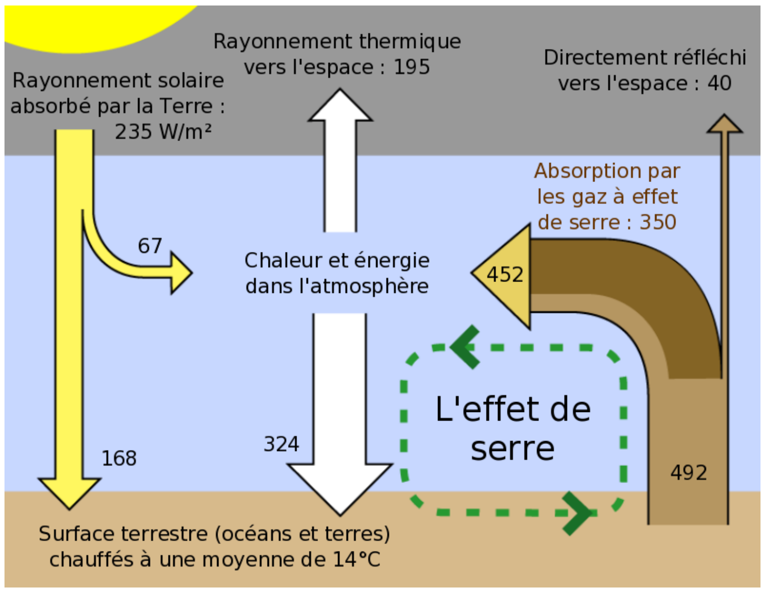


Figure 1: Schematic of the thermodynamics of climate. The figures are the real ones then not the estimations of the exercise. Picture from Wikipedia.

**2 Minimal Model of Climate** In this exercise we propose a small model to describe the climate. This model is composed of the Earth and an atmosphere receiving a power from the Sun. The three of them are modeled as black bodies. The Sun light reach the Earth and is partly reflected with a coefficient, the albedo, $a$. The Earth radiates toward the space but a fraction $b$ of this radiation is captured by the atmosphere. Finally, the atmosphere which radiate both toward the Earth and space.

1. We denote $T_A$ the temperature of the atmosphere and $T_E$ the temperature of the Earth. Write the thermal balances of the Earth and the atmosphere.
2. Solve these equations for $T_E$.
3. The albedo $a$ is depending on $T_E$, why?
4. Let us assume $a$ to be small for $T_E > T_f = 273$ K and large for $T_E < T_f$. Draw the diagram of the dynamics of $T_E$.
5. Give an estimate of $T_E$ using that the albedo is $a \approx 1/3$ and the greenhouse effect is $b \sim 0.9$. Discuss the climate change.
6. (Bonus) Make the entropic balance of the Earth.

**Correction**

1. We work by unit surface. The Earth receives a flux $p_{SE}$ from the Sun, a fraction $a$ of it being directly reflected toward space, and a flux $\sigma T_A^4$ from the atmosphere, and emits a flux $\sigma T_E^4$. Thus
$$(1-a)p_{SE} + \sigma T_A^4 = \sigma T_E^4. \tag{4}$$
Similarly the balance of the atmosphere is
$$b\sigma T_E^4 = 2\sigma T_A^4. \tag{5}$$
2. Thus, we have
$$(1-a)p_{SE} + \frac{b}{2}\sigma T_E^4 = \sigma T_E^4 \Longrightarrow T_E = \sqrt[4]{\frac{(1-a)p_{SE}}{\sigma(1-b/2)}}. \tag{6}$$
3. Because $a$ depends on the surface of ice and snow.
4. We assume the temperature of the atmosphere to remain almost contant during a perturbation. Thus,
$$\frac{M_E C_E}{\mathcal{A}_E}\partial T_E = (1-a(T_E))p_{SE} - (1-b/2)\sigma T_E^4. \tag{7}$$
We may draw the two curves and find 3 equilibrium points. The point $T_E = T_f$ is unstable and we have 2 stable states: the warm and glacial periods. In practice, $a$ is more subtile so that the glacial period has several equillibirum, some of them having a temperature above $T_f$. Typically, the last glacial period temperature was approximately 280 K.
5. We calculate $T_E \approx 291$ K. In reality, $T_E \approx 287$ K, so our model is pretty good. The climate change is an accumulation of greenhouse gases which boost $b$. In the limit $b = 1$, we would get a temperature $T_E \approx 298$ K. Thus, we may a temperature rise of $\approx 7$ K which is slightly more than the difference between our period and the last glacial period.

**3 Hydropower** We now estimate the resources in hydropower. We denote $\phi$ the height of water which falls in average and by unit time and unit surface. We consider the cycle of water.

1. The whole cycle originates from the solar radiations. What is the power used to generate this cycle of water? Give an estimate.
2. This water is then stored in clouds, thus this energy becomes a potential energy. Estimate this potential energy flux and the efficiency of the cycle of water.
3. Estimate the Carnot efficiency and the Curzon-Ahlborn efficiency to comment this result.
4. Give an estimate of the resources in hydropower. For comparison is global hydroelectricity production is ≈ 1.3 TW and the global energy consumption is ≈ 18 TW.
5. The average height of France is 375 m. Estimate the hydropower resources of France. For comparison, the energy consumption in France is ≈ 300 GW and the solar energy production is ≈ 25 GW.

**Correction**

1. We need to evaporate all this water to launch the cycle. This requires a power $\mathcal{P}_v = 4\pi R_E^2 \phi \rho_w l_v$ where $l_v \approx 2.25 \cdot 10^6$J/kg is the vaporization energy and $\rho_v$ is the density of water. Typically, $\phi \sim 1$ m/an $\cdot$ m$^2$. We then estimate $\mathcal{P}_v \sim 3.7 \cdot 10^{16}$ W, which is $\sim 20\%$ of the received power $\mathcal{P}_{SE}$.
2. The clouds are typically at a height $H \sim 5$ km. Thus, the potential energy flux is $\mathcal{P}_c = 4\pi R_E^2 \phi \rho_w g H \sim 8 \cdot 10^{14}$ W. The efficiency is then
$$\eta = \frac{\mathcal{P}_v}{\mathcal{P}_c} \sim 2\%. \tag{8}$$
3. From the previous exercise we have seen $T_E \approx 290$ K and $T_A \approx \sqrt[4]{b/2}T_E \approx 240$ K. Actually, for the considered height, it is more $T_A \approx 250$ K. We then calculate
$$\eta_C = 1 - \frac{T_A}{T_E} \approx 14\% \qquad \eta_{CA} = 1 - \sqrt{\frac{T_A}{T_E}} \approx 7\%. \tag{9}$$
Conclusion the cycle of water is very irreversible and thus not very efficient.
4. We use that $1/3$ of the Earth surface is made of land and the average height of these lands is $h \approx 400$ m. Thus, the available hydropower is
$$\mathcal{P}_c = 4\pi R_E^2 \phi \rho_w g h/3 \sim 2 \cdot 10^{13} \text{ W} = 20 \text{ TW}. \tag{10}$$
In practice not all of this energy could be used and the hydropower is almost at its maximum.
5. Similarly for France, the available hydropower is
$$\mathcal{P}_c = \mathcal{A}_{France} \phi \rho_w g h \sim 64 \text{ GW}. \tag{11}$$
In practice not all of this energy could be used and the hydropower is almost at its maximum.

**3 Wind power** We now estimate the resources in wind.

1. Calculate the mass of the atmosphere.
2. We assume the atmosphere to be homogeneous. What is the kinetic energy of the winds? Give an estimate.
3. What is the typical time for the change of winds?
4. Give an estimate of the resources in wind power. For comparison is global wind power production is ≈ 0.2 TW and the global energy consumption is ≈ 18 TW.
5. Estimate the hydropower resources of France taking the hypothesis of covering 1% of the surface of wind turbines. For comparison, the energy consumption in France is ≈ 300 GW and the wind power production is ≈ 5 GW.

**Correction**

1. The weight of the atmosphere is compensated by the pressure force at the surface. Thus, $M_A = 4\pi R_E^2 \frac{P_0}{g} \approx 5 \cdot 10^{18}$ kg.
2. The kinetic energy is $E_k = \frac{1}{2} M_A \bar{u}^2$ where $\bar{u}$ is the average velocity of winds. We may propose $\bar{u} \sim 20$ m/s. We then deduce $E_k \sim 10^{20}$ J.
3. The cycle day-night has a crucial influence on the winds so we may use $\tau \sim 1$ day.
4. We estimate $\mathcal{P}_w \sim E_k/\tau \sim 10^{15}$ W $\sim 1000$ TW.
5. We estimate $\mathcal{P} \sim \frac{\mathcal{A}_{France}/100}{\mathcal{A}_E} \mathcal{P}_w \sim 10^{10}$ W $\sim 10$ GW.